MathML Formatting (with $\TeX$ Rules and $\TeX$ Fonts)

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HELM Project http://helm.cs.unibo.it

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W3C Math Working Group http://www.w3.org/Math

Summary

- MathML
- Definition of the problem
- Architecture of the formatting engine
- Area model for MathML
- MathML formatting rules
- Conclusion

MathML Presentation: example

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```
<math xmlns="http://www.w3.org/1998/Math/MathML">
```

$$\lim_{x \to 0} \frac{\sin x}{x} = 25$$

```
</math>
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    <mrow>









    </mrow>
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  </mrow>
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      <munder>
        <mo> lim </mo>
        <mrow>



        </mrow>
      </munder>
      <mfrac>
        <mrow>



        </mrow>
        <mi> x </mi>
      </mfrac>
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        <mrow>
          <mi> x </mi>
          <mo> &RightArrow; </mo>
          <mn> 0 </mn>
        </mrow>
      </munder>
      <mfrac>
        <mrow>
          <mi> sin </mi>
          <mo> &ApplyFunction; </mo>
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- Using presentation markup, it's possible to specify how an expression will look when displayed

MathML Presentation Summary

- tokens (`mi`, `mo`, `mn`)
- general layout schemata (`mfrac`, `msqrt`)
- scripts and limits (`msub`, `msup`, `munder`, `mover`)
- tables and alignment (`mtable`, `mtr`, `mtd`)
- style and attribute inheritance (`mstyle`)
- "live" expressions (`maction`)

There is a fair amount of semantics even in presentation elements:

- refine formatting, higher quality
- "meaningful" presentation (conversions)

$\TeX$ Formatting Rules: Appendix G

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About fractions:

If $C > T$, set $u \leftarrow \sigma_8$ and $v \leftarrow \sigma_{11}$. Otherwise set $u \leftarrow \sigma_9$ or σ_{10}, according as $\theta \neq 0$ or $\theta = 0$, and set $v \leftarrow \sigma_{12}$. . .

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About scripts:

If the translation of the nucleus is a character box, possibly followed by a kern, set u and v equal to zero; otherwise set $u \leftarrow h - q$ and $v \leftarrow d + r$, where h and d are the height and depth of the translated nucleus, and where q and r are the values of σ_{18} and σ_{19} in the font corresponding to styles $C\uparrow$ and $C\downarrow \ldots$

MathML Formatting

Two contrasting objectives:

- we want to design a MathML formatting engine which is capable of using TeX fonts (hence TeX formatting rules, see Appendix G)
- we don't want to tie the engine to TeX fonts, as we recognize that a significant part of the formatting process is independent of the environment

Plan

Deeper analysis of the formatting process:

- understand and separate the modules of the architecture that depend on the environment from those that don't
- define a common interface for the loss-less communication of information among the modules
- exploit the semantically rich MathML markup for refining the outcome automatically

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$$[\![\texttt{<}t\texttt{>}\ c_1 \cdots c_n\ \texttt{</}t\texttt{>}]\!] = \mathbf{f}_t(c_1 \cdots c_n)$$

$$[\![\texttt{<}t\texttt{>}\ X_1 \cdots X_m\ \texttt{</}t\texttt{>}]\!] = \mathbf{f}_t([\![X_1]\!], \ldots, [\![X_m]\!])$$

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$$[\![\texttt{<}t\texttt{>}\ c_1 \cdots c_n\ \texttt{</}t\texttt{>}]\!]_C = \mathbf{f}_t(C', c_1 \cdots c_n)$$

$$[\![\texttt{<}t\texttt{>}\ X_1 \cdots X_m\ \texttt{</}t\texttt{>}]\!]_C = \mathbf{f}_t(C', [\![X_1]\!]_{C_1}, \ldots, [\![X_m]\!]_{C_m})$$

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$$
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[\![\texttt{<}t\texttt{>}\; X_1 \cdots X_m \;\texttt{</}t\texttt{>}]\!]_C &= \mathbf{f}_t(C', [\![X_1]\!]_{C_1}, \ldots, [\![X_m]\!]_{C_m})
\end{aligned}
$$

We seek for a decomposition of the $\mathbf{f}_t$ functions such that

- $\mathbf{f}_t \approx \mathbf{g}_t \circ \mathbf{h}_t$
- the $\mathbf{g}_t$ are independent of the environment
- neither of $\{ \mathbf{g}_t \}$ and $\{ \mathbf{h}_t \}$ is trivial

Criteria for decomposition

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- any operation that depends only on the "size" (the bounding box) of the components is part of the $\mathbf{g}_t$

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- any operation thay may depend on the "shape" (the content) of the components is part of the $\mathbf{h}_t$

  Example: Unicode strings, scripts, accents, fractions...

Areas

Formatting functions return areas that describe glyphs and their geometrical relationship

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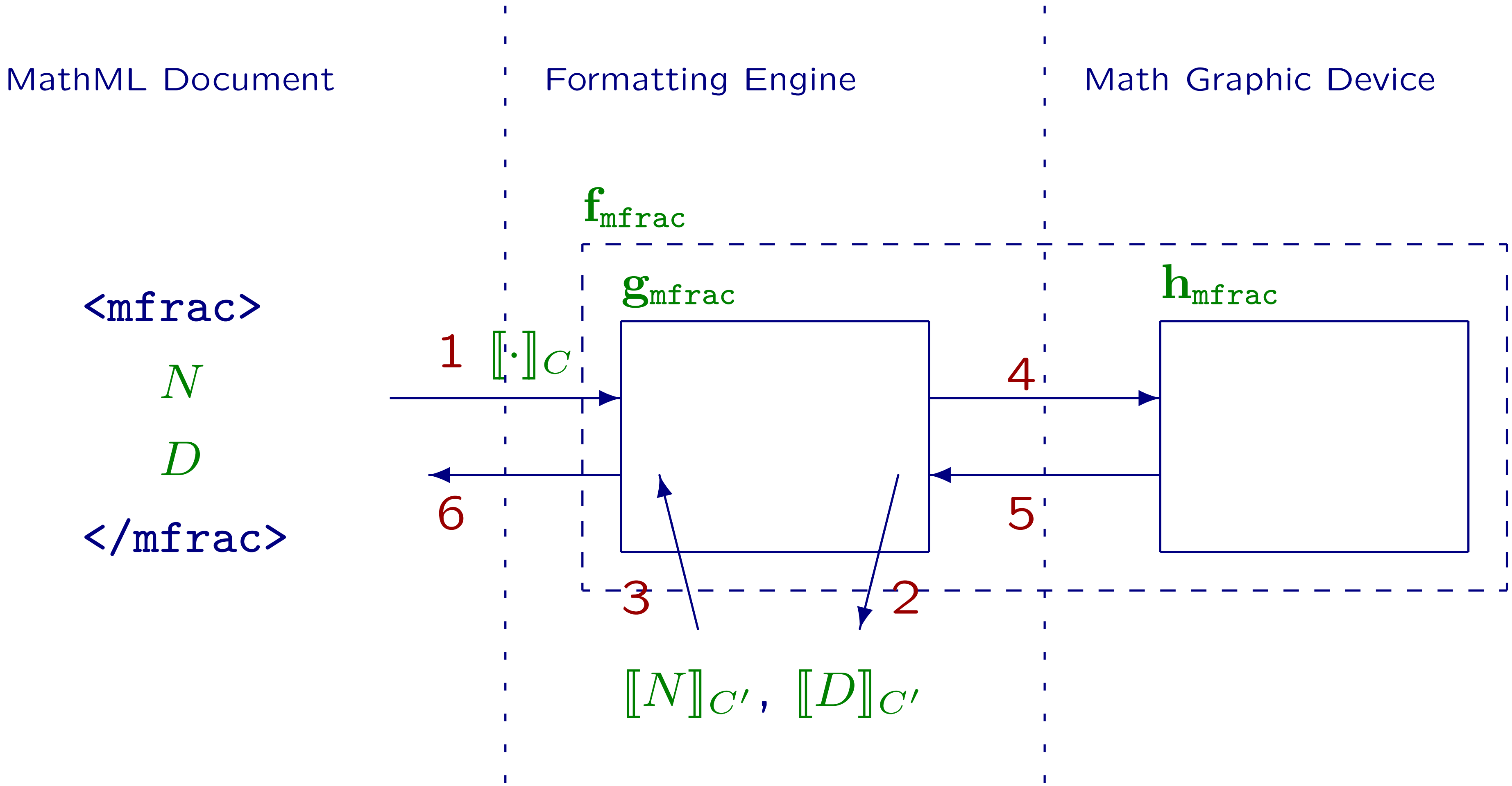
Areas are opaque to the formatting engine: the formatting engine can ask areas for a (very limited) set of properties, but doesn't know anything about their type

Areas are transparent to the graphic device for math: the device can do different things depending on the type and the content of areas

Example



MathML Document
Formatting Engine
Math Graphic Device
$\mathbf{f}_{\texttt{mfrac}}$
$\mathbf{g}_{\texttt{mfrac}}$
$\mathbf{h}_{\texttt{mfrac}}$
<mfrac>
N
D
</mfrac>
1 $[\![\cdot]\!]_C$
4
5
6
3
2
$[\![N]\!]_{C'}$, $[\![D]\!]_{C'}$

Areas

Requirements of the area model:

- language that we can use to express the layout of a mathematical formula in a concise and unambiguous way
- language that can be extended such that new types of entities can be introduced by an implementation (e.g. selection, back-pointers,...)

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An area describes a rectangular portion of the output medium as a tree structure. Each node in the tree (an area node) is identified by:

- a type
- a possibly empty set of properties
- a possibly empty, ordered list of child areas

Areas as TEX boxes

Area	TEX	Description
$G[\cdot]$		Glyph
K_n	`\kern`	Kern with value n. The kern is horizontal or vertical depending on the container it is in
F	`\hfill \vfill`	Filler area
R_n	`\hrule \vrule`	Filler rule of thickness n
$S_n[\alpha]$	`\raisebox` `\lowerbox`	Shift α's baseline by n
$H[\alpha_1, \ldots, \alpha_n]$	`\hbox`	Horizontal group of areas α_1,. . . ,α_n
$V_k[\alpha_1, \ldots, \alpha_n]$	`\vbox`	Vertical group of areas α_1,. . . ,α_n, where α_k is the reference area that determines the baseline

$[\![$`<mfrac>` N D `</mfrac>`$]\!]_C$

Formatting context update:

$$
\begin{aligned}
C' &= C\,[\mathit{element} \leftarrow \mathit{this}] \\
C'' &= C'[\text{if } C'.\mathit{displayStyle} = \text{true then} \\
&\qquad\qquad \mathit{displayStyle} \leftarrow \text{false} \\
&\qquad \text{else} \\
&\qquad\qquad \mathit{scriptLevel} \leftarrow C'.\mathit{scriptLevel} + 1; \\
&\qquad\qquad \mathit{aSize} \leftarrow C'.\mathit{aSize} \times C'.\mathit{sizeMult}; \\
&\qquad\qquad \mathit{size} \leftarrow \max\{C'.\mathit{minSize}, C'.\mathit{aSize} \times C'.\mathit{sizeMult}\}]
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\end{aligned}
$$

Fraction formatting:

$$
\begin{aligned}
& [\![\texttt{<mfrac>}\ N\ D\ \texttt{</mfrac>}]\!]_C \\
= \quad & \mathbf{h}_{\texttt{mfrac}}(C', H[F, [\![N]\!]_{C''}, F], H[F, [\![D]\!]_{C''}, F]) \qquad \text{Rules 15-15d} \\
= \quad & S_a[V_3[H[F, [\![D]\!]_{C''}, F], K_d, R_h, K_n, H[F, [\![N]\!]_{C''}, F]]]
\end{aligned}
$$

$[\![$ <msubsup> A B C </msubsup> $]\!]_C$

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\begin{aligned}
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&\qquad size \leftarrow \max\{C'.minSize, C'.aSize \times C'.sizeMult\}]
\end{aligned}
$$

Script formatting:

$$
\begin{aligned}
& [\![\texttt{<msubsup>}\ A\ B\ C\ \texttt{</msubsup>}]\!]_C \\
= \quad & \mathbf{h}_{\texttt{script}}(C', [\![A]\!]_{C'}, [\![B]\!]_{C''}, [\![C]\!]_{C''}) \qquad \text{Rules 18-18f} \\
= \quad & H[[\![A]\!]_{C'}, S_{-b}[V_1[H[K_{-d_1}, [\![B]\!]_{C''}], K_s, H[K_{d_2}, [\![C]\!]_{C''}]]]]
\end{aligned}
$$

More on scripts

Mere knowledge of the base element's bounding box isn't always enough for placing scripts correctly:

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$$\int$$

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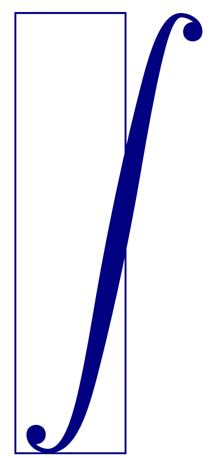

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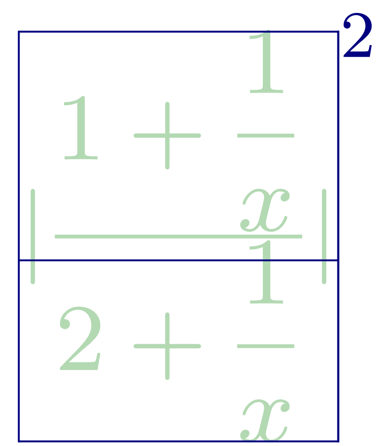

It is fundamental to communicate information on the type and content of areas when these must be combined together

Tricky $\TeX$ (I)

$\TeX$ is able to work things out with little effort. . .

$$|\frac{1+\frac{1}{x}}{2+\frac{1}{x}}|^2$$

Tricky TEX (I)

TEX is able to work things out with little effort. . .

$$\left|\frac{\displaystyle 1+\frac{1}{x}}{\displaystyle 2+\frac{1}{x}}\right|^2$$

. . . because there is little concern about structure:

```
|\frac{\displaystyle1+\frac{1}{x}}
      {\displaystyle2+\frac{1}{x}}|^2
```

Tricky $\TeX$ (II)

`\sin x`	$\sin x$	`\sin(x)`	$\sin(x)$

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`\sin x` $\sin x$ `\sin(x)` $\sin(x)$

TEX solves this problem by defining the `sin` macro as

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A MathML formatter can address the problem more generally:

```
<mrow>
  <mi> sin </mi>
  <mo> &#x2061; </mo>
  <mi> x </mi>
</mrow>
```

```
<mrow>
  <mi> sin </mi>
  <mo> &#x2061; </mo>
  <mrow>
    <mo> ( </mo>
    <mi> x </mi>
    <mo> ) </mo>
  </mrow>
</mrow>
```

Implementation(s)

Two implementations:

- PocketMath

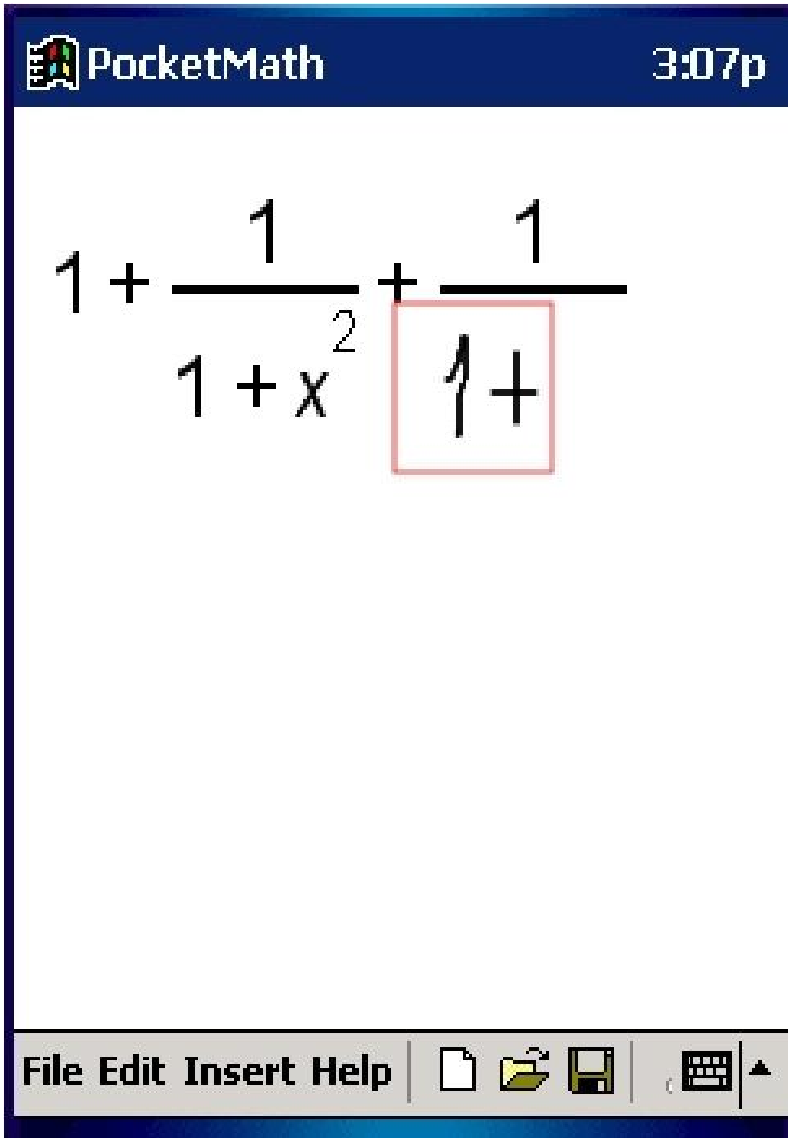
  (Maple)

- GtkMathView

  (GPL, `http://helm.cs.unibo.it/mml-widget/`)

MathML in hand-held devices

(with Stephen M. Watt, Ontario Research Centre for Computer Algebra)



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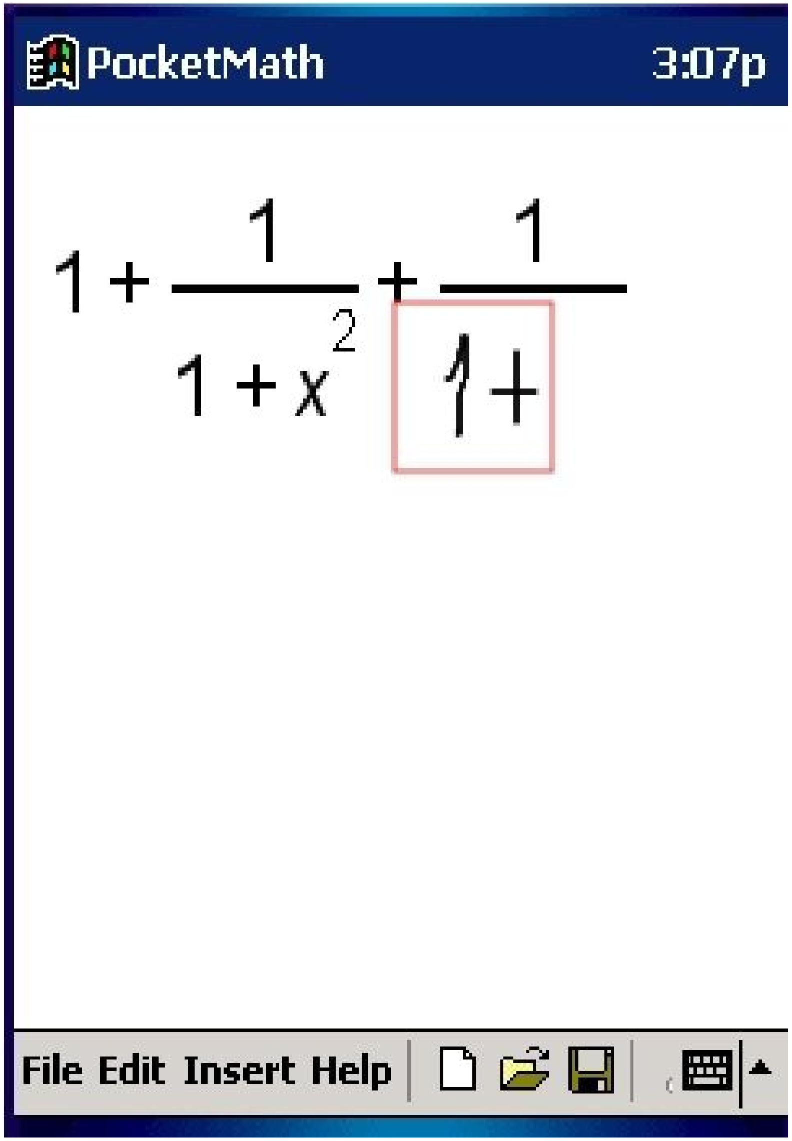

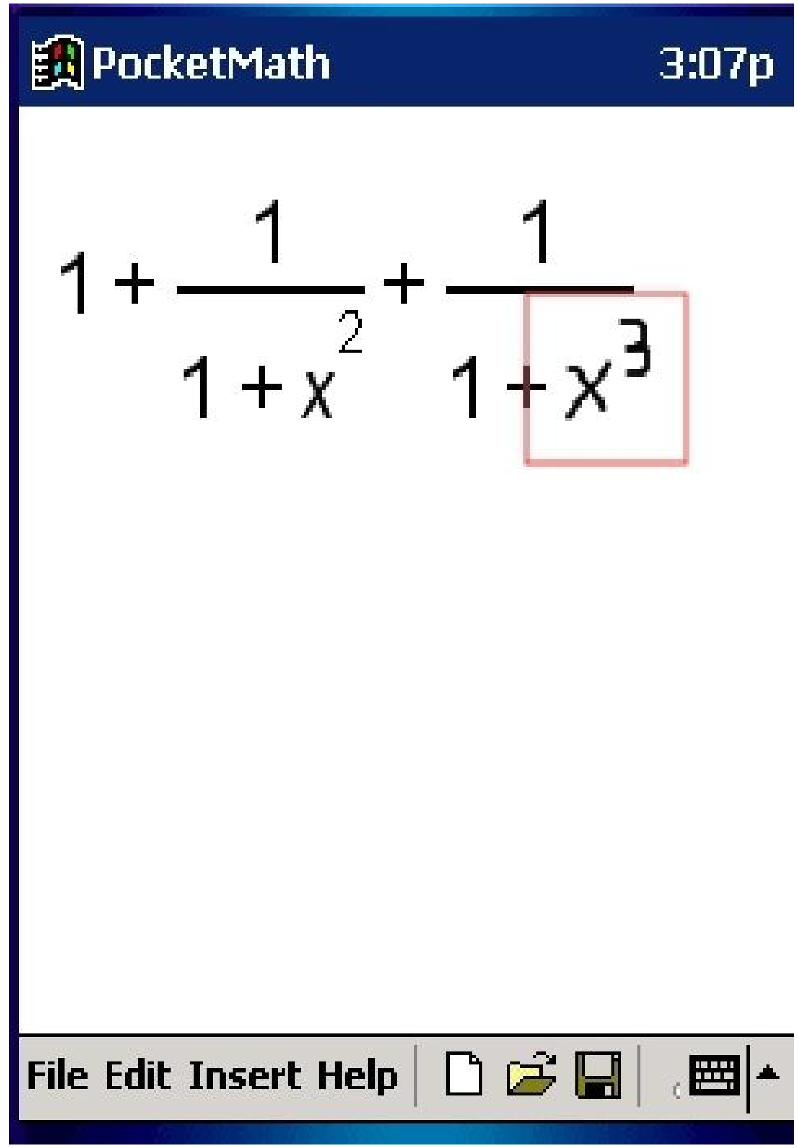

Wiley encyclopedias and textbooks

(with John Pedersen, John Wiley & Sons, Inc.)

- Burger's Medicinal Chemistry and Drug Delivery (Abraham)
- Encyclopedia of Catalysis (Horvath)
- Encyclopedia of Smart Materials (Schwartz)
- Encyclopedia of Software Engineering (Marciniak)
- Encyclopedia of Polymer Science and Technology
- Handbook of Chemicals and Gases for the Semiconductor Industry (Misra)
- Occupational Toxicants and MAK Values (Deutsche Forschungsgemeinschaft)
- Stevens' Handbook of Experimental Psychology (Pashler)
- Textbook of Biochemistry (Devlin)
- Ullmann's Encyclopedia of Industrial Chemistry (German branch of Wiley)

Also

- a number of Higher Ed/College textbooks being processed

Math in TEX + TEX fonts

- TEX is a single-minded system that does a very good job in a fixed environment: quality printing on paper with TEX's fonts
- TEXfonts have built-in "intelligence"
- any other system that uses TEXfonts will have to understand that "intelligence" and handle it correctly (see Mozilla)
- because TEX has a "standard" implementation, tweaks in TEX markup are harmless

MathML

- MathML can be formatted using TeX formatting rules and also TeX fonts
- it is possible to design the formatter so that it can be adapted very easily for different environments
- because of the semantically rich markup, a smart MathML formatter can handle automatically cases that need author's assistance in TeX (tweaks, line-breaking)