This drawing procedure takes shape from the macros \texttt{\textbackslash pslines}, \texttt{\textbackslash pspolygon}, \texttt{\textbackslash multiplo} and \texttt{\textbackslash parametricplot} in PStricks\textsuperscript{1}. The \TeX arithmetic gives the pointwise parametrization of curves.

\section{How to make a boundary using Bézier curves}

We can make the closed boundary of a plane domain by joining Bézier curves. This can produce various boundaries. We consider here two Bézier curves of degree 5 whose vector functions are

\begin{align*}
r_a(t) = & \sum_{i=0}^{5} \binom{5}{i} t^i (1-t)^{5-i} a_i, \\
r_b(t) = & \sum_{i=0}^{5} \binom{5}{i} t^i (1-t)^{5-i} b_i,
\end{align*}

where $a_i$ and $b_i$ are the position vectors of control points $M_i$, $N_i$ of the two curves, respectively, $i = 0, \ldots, 5$. Here we choose: $M_0 = (0, 0)$, $M_1 = (0, 1.5)$, $M_2 = (1.5, 1)$, $M_3 = (3, 2)$, $M_4 = (5, 1.5)$, $M_5 = (5, 0)$, and $N_0 = (5, 0)$, $N_1 = (5, -1.5)$, $N_2 = (3.5, -1)$, $N_3 = (2, -2)$, $N_4 = (0, -1.5)$, $N_5 = (0, 0)$. The two curves and their control points are shown in Figure 1 and the picture in it gives us an obvious explanation of how to combine two Bézier curves to make a closed and smooth boundary. To change the shape of the boundary, we take one couple of multipliers for the second

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Bezier_boundary}
\caption{A Bézier-spline boundary.}
\end{figure}

coordinates of $M_2$, $M_3$ and another one for those of $N_2$, $N_3$. To draw the curves $r_a(t)$ and $r_b(t)$ we

\textsuperscript{1}PStricks is the original work of Timothy Van Zandt (email address: tvz@econ.insead.fr). It is currently edited by Herbert Voß (hvoss@tug.org).
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need their parametrization \((X_a(t), Y_a(t))\) and \((X_b(t), Y_b(t))\), \(t \in [0, 1]\). From their control points and
the given couple of multipliers \(\alpha_M, \beta_M\), we derive
\[
\begin{align*}
X_a(t) &= 15t^2(1-t)^3 + 30t^3(1-t)^2 + 25t^4(1-t) + 5t^5, \\
Y_a(t) &= 7.5t(1-t)^4 + 10\alpha_M t^2(1-t)^3 + 20\beta_M t^3(1-t)^2 + 7.5t^4(1-t).
\end{align*}
\]
Similarly, we have
\[
\begin{align*}
X_b(t) &= 5(1-t)^5 + 25t(1-t)^4 + 35t^2(1-t)^3 + 20t^3(1-t)^2, \\
Y_b(t) &= -7.5t(1-t)^4 - 10\alpha_N t^2(1-t)^3 - 20\beta_N t^3(1-t)^2 - 7.5t^4(1-t),
\end{align*}
\]
where \(\alpha_N, \beta_N\) are multipliers. Then, these curves will be depicted by the macro \texttt{\textbackslash parametricplot}
with the corresponding declarations in the algebraic form:

\[
\begin{align*}
&\texttt{\def\XoneFive{15*t^2*(1-t)^3+30*t^3*(1-t)^2+25*t^4*(1-t)+5*t^5}} \\
&\texttt{\def\YoneFive#1#2%}{7.5*t*(1-t)^4+(#1)*10*t^2*(1-t)^3+(#2)*20*t^3*(1-t)^2+7.5*t^4*(1-t)} \\
&\texttt{\def\XtwoFive{5*(1-t)^5+25*t*(1-t)^4+35*t^2*(1-t)^3+20*t^3*(1-t)^2}} \\
&\texttt{\def\YtwoFive#1#2%}{(-7.5)*t*(1-t)^4+(#1)*(-10)*t^2*(1-t)^3+(#2)*(-20)*t^3*(1-t)^2+(#1)*(-7.5)*t^4*(1-t)}
\end{align*}
\]

where \#1, \#2 stand for \(\alpha_M\) and \(\beta_M\) in the definition of \texttt{\YoneFive} and for \(\alpha_N\) and \(\beta_N\) in that of
\texttt{\YtwoFive}, respectively. In Figure 2, the boundaries are drawn by using together the commands

\[
\texttt{\parametricplot[algebraic,plotpoints=200,linewidth=0.5pt]{0}{1}} \{\XoneFive[\alphaM]{\betaM}\} \\
\texttt{\parametricplot[algebraic,plotpoints=200,linewidth=0.5pt]{0}{1}} \{\YtwoFive[\alphaN]{\betaN}\}
\]

where, \texttt{\alphaM}, \texttt{\betaM}, \texttt{\alphaN} and \texttt{\betaN} are chosen values of \(\alpha_M, \beta_M, \alpha_N\) and \(\beta_N\), respectively.

![Figure 2](image)

Figure 2: From left to right, corresponding to the couple of values: \(\alpha_M = -0.5, \beta_M = 2.1, \alpha_N = 2.4, \beta_N = -0.4; \alpha_M = 1.1, \beta_M = 1.4, \alpha_N = 1.2, \beta_N = 1.5; \alpha_M = 1.7, \beta_M = 0.2, \alpha_N = 0.1, \beta_N = 1.8\)

2. How to draw a partition of a plane domain

To draw a partition of a plane domain whose boundary is made by the way that has just been
described, we need the two procedures \texttt{\NetDrawOne} and \texttt{\NetDrawTwo}, corresponding to \(r_a(t)\) and
\(r_b(t)\). Their calling sequences take three arguments in order: one for \(c\) (the number of cells) and the
others for \(\alpha_M\) and \(\beta_M\) or \(\alpha_N\) and \(\beta_N\). Both of \texttt{\NetDrawOne} and \texttt{\NetDrawTwo} are used here to find
2. How to draw a partition of a plane domain

Cells in a given grid that have points in common with \( r_a(t) \) and \( r_b(t) \), and to color those cells. To obtain the definition of these procedures, we first state the problem that we are considering here.

A grid of rectangular cells will be put on a rectangle \( R \) containing a domain \( D \) whose boundary consists of \( r_a(t) \) and \( r_b(t) \), including the given values of \( c, \alpha_M, \beta_M, \alpha_N \) and \( \beta_N \). Then, the steps for drawing a partition of \( D \) can be listed as follows:

- Coloring cells that have points in common with the boundary of \( D \). This is the result of calling \( \text{NetDrawOne}(c \{ \alpha_M \} \{ \beta_M \}) \) and \( \text{NetDrawTwo}(c \{ \alpha_N \} \{ \beta_N \}) \) together.
- Drawing the grid on \( R \), which has been chosen as \( R = \{(x, y): -1 \leq x \leq 6, -3 \leq y \leq 3\} \).
- Drawing the two curves \( r_a(t) \) and \( r_b(t) \).

In practice, a partition of \( D \) can be given by a calling sequence that has the following structure inside the \begin{pspicture} \end{pspicture} environment:

<table>
<thead>
<tr>
<th>Commands</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>\NetDrawOne{c}{\alpha_M}{\beta_M}\NetDrawTwo{c}{\alpha_N}{\beta_N}</td>
<td>Coloring cells</td>
</tr>
<tr>
<td>\multido{\nx=-1.00+\decimal\Xsize}{c+1}{\nx}</td>
<td>Drawing the grid on ( R )</td>
</tr>
<tr>
<td>\psline<a href="%5Cnx,-3">linewidth=0.2pt</a>(\nx,3)</td>
<td></td>
</tr>
<tr>
<td>\psline<a href="-1,%5Cny">linewidth=0.2pt</a>(6,\ny)</td>
<td></td>
</tr>
<tr>
<td>\parametricplot[algebraic,plotpoints=200,linecolor=white,linewidth=0.5pt]{0}{1}{\XoneBFive</td>
<td>\YoneBFive{\alpha_M}{\beta_M}}</td>
</tr>
<tr>
<td>\parametricplot[algebraic,plotpoints=200,linecolor=white,linewidth=0.5pt]{0}{1}{\XtwoFive</td>
<td>\YtwoFive{\alpha_N}{\beta_N}}</td>
</tr>
</tbody>
</table>

In the following, beside the definitions of \( \XoneFive \), \( \YoneFive \), \( \XtwoFive \) and \( \YtwoFive \), we list all the remaining macros by their control sequences only and the results derived from running them. They form the whole drawing package, and of course they are also put in the preamble or in a single \TeX\ file to be loaded when running the package.

- \def\xch: Setting \catcode‘p=12, \catcode‘t=12.
- \def\ych: Setting \catcode‘p=11, \catcode‘t=11.
- \def\decimal#1: Getting the numeric value of \#1 without unit.
- \def\FACTORIAL#1: The factorial function. Ex: \FACTORIAL{3} = 6.
- \def\BINOMIAL#1#2: Binomial coefficients. Ex: \BINOMIAL{5}{2} = 10.
- \def\BERNSTEIN#1#2#3: Bernstein’s functions. Ex: \BERNSTEIN{3}{2}{t} = \binom{5}{3} t^2 (1-t)^3.
- \def\XoneBC#1, \def\XtwoBC#1: The values of \( X_a(t) \), \( X_b(t) \). Ex: \XoneBC{0.5} = X_a(0.5).
- \def\YoneBC#1#2#3, \def\YtwoBC#1#2#3: The value of \( Y_a(t) \) with given values of \( \alpha_M, \beta_M \), and the value of \( Y_b(t) \) with given values of \( \alpha_N, \beta_N \). Ex: \YoneBC{0.5}{1.8}{1.2} = Y_a(0.5), with \( \alpha_M = 1.8, \beta_M = 1.2 \).
2. How to draw a partition of a plane domain

We recall here the main idea for this drawing procedure. We have already the expressions of $r_a(t)$ and $r_b(t)$ to draw these Bézier curves by the macro \parametricplot. But, we need to have in hand their points' coordinates to determine cells of a grid containing points in common with these curves. Fortunately, we may use the \TeX arithmetic to design expressions of $X_a(t)$, $Y_a(t)$, $X_b(t)$, and $Y_b(t)$, hence we can evaluate their values at each given $t \in [0, 1]$. For instance, the macro \def\XoneBC1 is destined for evaluating the value of $X_a(t)$, and since it is a linear combination of Bernstein’s functions, we need binomial coefficients. We do not take directly numeric values of those coefficients, and we have had the macro \def\BINOMIAL1#2 do that instead. It might be possible to use this macro in other problems later. In Appendix A, we list the replacement texts of the above macros and all the local and global variables they need.

From the listed macros, \NetDrawOne may now have its definition as

```latex
\def\NetDrawOne#1#2#3{\% \newdimen\Xsize \newdimen\Ysize \newdimen\tempx \newdimen\tempy \Xsize=7pt \divide\Xsize by #1 \Ysize=6pt \divide\Ysize by #1 \parametricplot[algebraic,fillstyle=solid,fillcolor=yellow!80,plotpoints=200, linewidth=0.5pt]{0}{1}{\XoneFive|\YoneFive{#2}{#3}} \multido{\nz=0.00+0.005}{200}{\XoneBC{\nz}\YoneBC{\nz}{#2}{#3}} \multido{\nx=-1.00+\decimal\Xsize}{#1}{\tempx=\nx pt\multido{\ny=-3.00+\decimal\Ysize}{#1}{\tempy=\ny pt \ifdim\YBST<\tempy\relax\else\advance\tempy by \Ysize\fi \ifdim\XBST<\tempx\relax\else\advance\tempx by \Xsize\fi \ifdim\YBST>\tempx\relax\else\advance\tempx by \Xsize\fi \ifdim\XBST>\tempy\relax\else\advance\tempy by \Ysize\fi \pspolygon[fillstyle=solid,fillcolor=blue!70, linecolor=black,linewidth=0.2pt]{\nx,\ny} ({\nx,\ny})} }
```

To obtain the definition of \NetDrawTwo, we just replace by \XtwoFive, \Ytwofive, \XtwoBC and \YtwoBC for \XoneFive, \YoneFive, \XoneBC and \YoneBC in that of \NetDrawOne, respectively.

Because of its inevitable shortcoming, \TeX often gives approximate results of calculations on dimensions. In the following table, let us see a slight difference between results obtained from the same expression (by declaration of operations) in Maple and in the \TeX arithmetic:

<table>
<thead>
<tr>
<th>$t$</th>
<th>0.12</th>
<th>0.33</th>
<th>0.54</th>
<th>0.74</th>
<th>0.97</th>
</tr>
</thead>
<tbody>
<tr>
<td>$XoneBC(t)$</td>
<td>1.9081</td>
<td>1.1921</td>
<td>2.6316</td>
<td>4.02428</td>
<td>4.98177</td>
</tr>
<tr>
<td>Maple</td>
<td>1.9203</td>
<td>1.1935</td>
<td>2.6328</td>
<td>4.02479</td>
<td>4.98266</td>
</tr>
</tbody>
</table>

Actually, calculations in determining if a point of the two curves belongs a cell almost give the desired result.

Before taking some examples, we give an explanation about what the main algorithm in the procedure \NetDrawOne (or \NetDrawTwo) is. Let us take an approximate sequence of points for the
2. How to draw a partition of a plane domain

A typical cell.

Figure 3: A typical cell.

curve $r_a(t)$, say $P_i(\text{XoneBC}\{t_i\}, \text{YoneBC}\{t_i\}\{\alpha_M\}\{\beta_M\}), i = 1, \ldots, 200$. Then, each $P_i$ is examined whether to be in a cell that has the reference point $C = (x, y)$ by the instruction that: do nothing if $P_i$ is

- below $d_3$, or
- above $d_4$, or
- on the left side of $d_1$, or
- on the right side of $d_2$;

otherwise, color the cell. This algorithm can be expressed in the form of

\[
\begin{align*}
  \ifdim\YBST<y \relax\else\text{advance } y \text{ by } ysize & \\
  \ifdim\YBST>y \relax\else & \\
  \ifdim\XBST<x \relax\else\text{advance } x \text{ by } xsize & \\
  \ifdim\XBST>x \relax\else & \\
  \pspolygon[fillstyle=solid,...]
\end{align*}
\]

where $\XBST$ and $\YBST$ hold the values of $\text{XoneBC}\{t_i\}$ and $\text{YoneBC}\{t_i\}\{\alpha_M\}\{\beta_M\}$, respectively. This structure is almost the same as that in the procedure $\text{NetDrawOne}$ (or $\text{NetDrawTwo}$), but the variables here are declared according to the gloss in Figure 3.

Finally, let us take two examples where we just give values of $c$ for the caption of figures. In Figure 4, we take the same couples $\alpha_M = 1.7$, $\beta_M = 0.2$, $\alpha_N = 0.1$ and $\beta_N = 1.8$ for its two pictures. In Figure 5, we take the same couples $\alpha_M = -0.5$, $\beta_M = 2.1$, $\alpha_N = 2.4$ and $\beta_N = -0.4$ for its three pictures.
A. Appendix: detailed definitions

The following is the list of all the detailed definitions needed for the procedure, except those of $\mathbf{XoneFive}$, $\mathbf{YoneFive}$, $\mathbf{XtwoFive}$, $\mathbf{YtwoFive}$, $\mathbf{NetDrawOne}$ and $\mathbf{NetDrawTwo}$.

\begin{verbatim}
\def\xch{\catcode\p=12 \catcode\t=12}
\def\ych{\catcode\p=11 \catcode\t=11}\xch
\def\dec#1pt{#1}\ych
\def\decimal#1{\expandafter\dec \the#1}
\newcount\Fa\newcount\Fct\newcount\tempA
\def\Factor{\ifnum\Fa=1\relax\else\advance\Fa by -1\multiply\Fct by \Fa\Factor\fi}
\def\FACTORIAL#1{\Fa=#1 \ifnum\Fa=0 \Fct=1\relax\else\Fct=\Fa\Factor\fi\global\tempA=\Fct}
\newcount\BINOM\newcount\temp\newcount\tmp\def\BINOMIAL#1#2{\%\temp=#1\advance\temp by -#2\FACTORIAL{#1}\tmp=\tempA\FACTORIAL{\temp}\temp=\tempA\divide\tmp by \temp\FACTORIAL{#2}\temp=\tempA\divide\tmp by \temp\global\BINOM=\tmp}
\newdimen\Xa\newdimen\Yb\newcount\kc\newdimen\BSTemp\def\xmult#1{\ifnum\kc<#1\advance\kc by 1\Yb=\decimal\Xa\Yb\xmult{#1}\else\relax\fi}
\end{verbatim}

Figure 5: From left to right: $c = 25$, $c = 45$ and $c = 63$. 
A. Appendix: detailed definitions

\def\BERNSTEIN#1#2#3{\Xa=#3pt\kc=0\Yb=1pt\xmult{#1}\kc=0\Xa=-\Xa\advance\Xa by 1pt\xmult{#2}\BINOMIAL{5}{#1}\global\BSTemp=\BINOMYb}
\newdimen\Xrf\newdimen\Yrf\newdimen\XoneBST\newdimen\YoneBST\newdimen\XtwoBST\newdimen\YtwoBST\newdimen\XBST\newdimen\YBST
\def\XoneBC#1{\BERNSTEIN{2}{3}{#1}\Xrf=1.5pt\XoneBST=\decimal\BSTemp\Xrf\BERNSTEIN{3}{2}{#1}\Xrf=3pt\advance\XoneBST by \decimal\BSTemp\Xrf\BERNSTEIN{4}{1}{#1}\Xrf=5pt\advance\XoneBST by \decimal\BSTemp\Xrf\BERNSTEIN{5}{0}{#1}\Xrf=5pt\advance\XoneBST by \decimal\BSTemp\Xrf\global\XBST=\XoneBST}
\def\XtwoBC#1{\BERNSTEIN{0}{5}{#1}\Xrf=5pt\XtwoBST=\decimal\BSTemp\Xrf\BERNSTEIN{1}{4}{#1}\Xrf=5pt\advance\XtwoBST by \decimal\BSTemp\Xrf\BERNSTEIN{2}{3}{#1}\Xrf=3.5pt\advance\XtwoBST by \decimal\BSTemp\Xrf\BERNSTEIN{3}{2}{#1}\Xrf=2pt\advance\XtwoBST by \decimal\BSTemp\Xrf\global\XBST=\XtwoBST}
\def\YoneBC#1#2#3{\BERNSTEIN{1}{4}{#1}\Yrf=1.5pt\YoneBST=\decimal\BSTemp\Yrf\BERNSTEIN{2}{3}{#1}\Yrf=1pt\Yrf=#2\Yrf\advance\YoneBST by \decimal\BSTemp\Yrf\BERNSTEIN{3}{2}{#1}}


References


