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17

On-line Tutorial on LaTeX

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This document is generated from LaTeX sources compiled with pdfLaTeX v. 14e in an INTEL Pentium III 700 MHZ system running Linux kernel version 2.2.14-12. The packages used are hyperref.sty and pdfscreen.sty

17 Typesetting Theorems

17.1 Theorems in LaTeX

In Mathematical documents we often have special statements such as *axioms* (which are nothing but the assumptions made) and *theorems* (which are the conclusions obtained, sometimes known by other names like *propositions* or *lemmas*). These are often typeset in different font to distinguish them from surrounding text and given a name and a number for subsequent reference. Such distinguished statements are now increasingly seen in other subjects also. We use the term *theorem-like statements* for all such statements.

LaTeX provides the declaration `\newtheorem` to define the theorem-like statements needed in a document. This command has two arguments, the first for *the name we assign to the environment* and the second, *the name to be printed with the statement*. Thus if you want

Theorem 1. *The sum of the angles of a triangle is* 180°.

you first specify

```
\newtheorem{thm}{Theorem}
```

and then type

```
\begin{thm}
The sum of the angles of a triangle is $180^\circ$.
\end{thm}
```

Note that in the command `\newtheorem` the first argument can be any name you fancy, instead of the `thm` given here. Also, it is a good idea to keep all your `\newtheorem` commands together in the preamble.

The `\newtheorem` command has a couple of optional arguments which control the way the corresponding statement is numbered. For example if you want the above theorem to be numbered 1.1 (the first theorem of the first section) rather than a plain 1, then you must specify

```
\newtheorem{thm}{Theorem}[section]
```

in the `\newtheorem` command. Then the same input as above for the theorem produces

Theorem 17.1.1. *The sum of the angles of a triangle is* 180°.

The next **Theorem** will be numbered 1.2, the third **Theorem** in the fourth section will be numbered 4.3 and so on.

The other optional argument of the `\newtheorem` command is useful when you have several different types of theorem-like statements (such as lemmas and corollaries) and you want some of them to share the same numbering sequence. For example if you want

Theorem 17.1.2. *The sum of the angles of a triangle is* 180°.

An immediate consequence of the result is the following

Corollary 17.1.3. *The sum of the angles of a quadrilateral is* 360°.

Then you must specify

```
\newtheorem{cor}[thm]{Corollary}
```

after the specification `\newtheorem{thm}[section]` and then type

```
\begin{thm}
The sum of the angles of a triangle is $180^\circ$.
\end{thm}

An immediate consequence of the result is the following
\begin{cor}
The sum of the angles of a quadrilateral  is $360^\circ$.
\end{cor}
```

The optional argument `thm` in the definition of the `cor` environment specifies that "Corollaries" and "Theorems" are to be numbered in the same sequence.

A theorem-like environment defined using the `\newtheorem` command has also an optional argument which is used to give a *note* about the theorem such as the name of its discoverer or its own common name. For example, to get

Theorem 17.1.4 (Euclid). *The sum of the angles of a triangle is* 180°.

you must type

```
\begin{thm}[Euclid]
The sum of the angles of a triangle is $180^\circ$.
\end{thm}
```

Note the optional argument `Euclid` after the `\begin{thm}`. This use of [...] for optional notes sometimes lead to unintended results. For example, to get

Theorem 17.1.5. $[0, 1]$ *is a compact subset of* $\mathbb{R}$.

if you type

```
\begin{thm}
[0,1] is a compact subset of $\mathbb{R}$.
\end{thm}
```

then you get

Theorem 17.1.6 (0,1). *is a compact subset of* $\mathbb{R}$.

Do you see what happened? The string 0,1 within [ ] at the beginning of the theorem is considered an optional note by LaTeX ! The correct way is to type

```
\begin{thm}
$[0,1]$ is a compact subset of $\mathbb{R}$.
\end{thm}
```

Now all the theorem-like statements produced above have the *same typographical form*— name and number in **boldface** and the body of the statement in *italics*. What if you need something like

Theorem 17.1.1 (Euclid). *The sum of the angles of a triangle is* 180°.

Such customization is necessitated not only by the aesthetics of the author but often by the whims of the designers in publishing houses also.

17.2 Designer Theorems— The amsthm package

The package amsthm affords a high level of customization in formatting theorem-like statements. Let's first look at the predefined *s2tyles* available in this package.

17.2.1 Ready made styles

The default style (this is what you get if you don't say anything about the style) is termed `plain` and it is what we have seen so far—name and number in boldface and body in italic. Then there is the `definition` style which gives name and number in boldface and body in roman. And finally there is the `remark` style which gives number and name in italics and body in roman.

For example if you put in the preamble

```
\usepackage{amsthm}
\newtheorem{thm}{Theorem}[section]
\theoremstyle{definition}
\newtheorem{dfn}{Definition}[section]
\theoremstyle{remark}
\newtheorem{note}{Note}[section]
\theoremstyle{plain}
\newtheorem{lem}[thm]{Lemma}
```

and then type somewhere in your document

```
\begin{dfn}
A triangle is the figure formed by joining each pair
of three non collinear points by line segments.
\end{dfn}

\begin{note}
A triangle has three angles.
\end{note}

\begin{thm}
The sum of the angles of a triangle is $180^\circ$.
\end{thm}

\begin{lem}
The sum of any two sides of a triangle is greater
than or equal to the third.
\end{lem}
```

then you get

Definition 17.2.1. A triangle is the figure formed by joining each pair of three non collinear points by line segments.

Note 17.2.1. A triangle has three angles.

Theorem 17.2.1. *The sum of the angles of a triangle is* 180°.

Lemma 17.2.2. *The sum of any two sides of a triangle is greater than or equal to the third.*

Note how the `\theoremstyle` command is used to switch between various styles, especially the last `\theoremstyle{plain}` command. Without it, the previous `\theoremstyle{remark}` will still be in force when `lem` is defined and so “Lemma” will be typeset in the `remark` style.

17.2.2 Custom made theorems

Now we are ready to roll our own “theorem styles”. This is done via the `\newtheoremstyle` command, which allows us to control almost all aspects of typesetting theorem like statements. this command has nine parameters and the general syntax is

```
\newtheoremstyle%
{⟨name⟩}%
{⟨abovespace⟩}%
{⟨belowspace⟩}%
{⟨bodyfont⟩}%
{⟨indent⟩}%
{⟨headfont⟩}%
{⟨headpunct⟩}%
{⟨headspace⟩}%
{⟨custom-head-spec⟩}%
```

The first parameter *name* is the name of the new *style*. Note that it is *not* the name of the *environment* which is to be used later. Thus in the example above `remark` is the name of a new style for typesetting theorem like statements and `note` is the name of the environment subsequently defined to have this style. (and `Note` is the name of the statement itself).

The next two parameters determine the vertical space between the theorem and the surrounding text—the *abovespace* is the space from the preceding text and the *belowspace* the space from the following text. You can specify either a rigid length (such as 12pt) or a rubber length (such as `\baselineskip`) as a value for either of these. Leaving either of these empty sets them to the “usual values”. (Technically the `\topsep`).

The fourth parameter *bodyfont* specifies the font to be used for the body of the theorem-like statement. This is to be given as a *declaration* such as `\scshape` or `\bfseries` and *not* as a *command* such as `\textsc` or `\textbf`. If this is left empty, then the main text font of the document is used.

The next four parameters refer to the *theoremhead*—the part of the theorem like statement consisting of the name, number and the optional note. The fifth parameter *indent* specifies the indentation of *theoremhead* from the left margin. If this is empty, then there is no indentation of the *theoremhead* from the left margin. The next parameter specifies the font to be used for the *theoremhead*. The comments about the parameter *bodyfont*, made in the previous paragraph holds for this also. The parameter *headpunct* (the seventh in our list) is for specifying the *punctuation* after the theoremhead. If you don’t want any, you can leave this empty. The last parameter in this category (the last but one in the entire list), namely *headspace*, determines the (horizontal) space to be left between the *theoremhead* and the *theorembody*. If you want only a normal interword space here put a *single blank space* as `{ }` in this place. (Note that it is not the same as leaving this *empty* as in `{}`). Another option here is to put the command `\newline` here. Then instead of a space, you get a linebreak in the output; that is, the *theoremhead* will be printed in a line by itself and the *theorembody* starts from the next line.

The last parameter *custom-head-spec* is for customizing *theoremheads*. Since it needs some explanation (and since we are definitely in need of some breathing space), let’s now look at a few examples using the eight parameters we’ve already discussed.

It’s almost obvious now how the last theorem in Section 1 (see Page 3) was designed. It was generated by

```
\newtheoremstyle{mystyle}{}{}{\slshape}%
{}{\scshape}{.}{ }{}

\theoremstyle{mystyle}
\newtheorem{mythm}{Theorem}[section]

\begin{mythm}
The sum of the angles of a triangle is $180^\circ$.
\end{mythm}
```

As another example, consider the following

```
\newtheoremstyle{mynewstyle}{12pt}{12pt}{\itshape}%
  {}{\sffamily}{:}{\newline}{}

\theoremstyle{mynewstyle}
\newtheorem{mythm}{Theorem}[section]

\begin{mynewthm}[Euclid]
The sum of the angles of a triangle is $180^\circ$.
\end{mynewthm}
```

This produces

Theorem 17.2.1 (Euclid):
The sum of the angles of a triangle is 180°.

Do you need anything more? Perhaps yes. Note that *theoremhead* includes the optional note to the theorem also, so that the font of the number and name of the theorem-like statement and that of the optional note are always the same. What if you need something like

Cauchy's Theorem (Third Version). *If G is a simply connected open subset of* $\mathbb{C}$, *then for every closed rectifiable curve* γ *in G, we have*

$$\int_\gamma f = 0$$

It is in such cases, that the last parameter of `\newtheoremstyle` is needed. Using it we can separately customize the name and number of the theorem-like statement and also the optional note. The basic syntax is for setting this parameter is

```
{commands#1commands#2commands#3}
```

where `#1` corresponds to the name of the theorem-like statement, `#2` corresponds to its number and `#3` corresponds to the optional note. We are here actually supplying the replacement text for a command `\thmhead` which has three arguments. It's as if we are defining

```
\renewcommand{\thmhead}[3]{...#1...#2...#3}
```

but without actually typing the `\renewcommand{\thmhead}[3]`. For example the theorem above (Cauchy's Theorem) was produced by

```
\newtheoremstyle{diffnotenonum}{}{}{\itshape}{}%
  {\bfseries}{.}{ }{#1 (\mdseries #3)}

\theoremstyle{diffnotenonum}
\newtheorem{Cauchy}{Cauchy's Theorem}

\begin{Cauchy}[Third Version]
If $G$ is a simply connected open subset of
$\mathbb{C}$, then for every closed rectifiable
curve $\gamma$ in $G$, we have
\begin{equation*}
\int_\gamma f=0
\end{equation*}
\end{Cauchy}
```

Note that the absence of `#2` in the *custom-head-spec*, suppresses the theorem number and that the *space* after `#1` and the command `(\mdseries#3)` sets the optional note in medium size within parentheses and with a preceding space.

Now if you try to produce

Riemann Mapping Theorem. *Every open simply connected proper subset of* $\mathbb{C}$ *is analytically homeomorphic to the open unit disk in* $\mathbb{C}$*.*

by typing

```
\theoremstyle{diffnotenonum}
\newtheorem{Riemann}{Riemann Mapping THeorem}

\begin{Riemann}
Every open simply connected proper subset of
$\mathbb{C}$ is analytically homeomorphic to
the open unit disk in $\mathbb{C}$.
\end{Riemann}
```

you'll get

Riemann Mapping Theorem (). *Every open simply connected proper subset of* $\mathbb{C}$ *is analytically homeomorphic to the open unit disk in* $\mathbb{C}$*.*

Do you see what's happened? In the `\theoremstyle{diffnotenonum}`, the parameter controlling the *note* part of the *theoremhead* was defined as `(\mdseries #3)` and in the `\newtheorem{`*Riemann*`}`, there is no optional note, so that in the output, you get an empty "note", *enclosed in parantheses* (and also with a preceding space).

To get around these difficulties, you can use the commands `\thmname`, `\thmnumber` and `\thmnote` *within* the {⟨*custom-head-spec*⟩} as

```
{\thmname{⟨commands#1⟩}%
    \thmnumber{⟨commands#2⟩}%
     \thmnote{⟨commands#3⟩}
```

Each of these three commands will typeset its argument *if and only if the corresponding argument in the* `\thmhead` *is non empty*. Thus the correct way to get the **Riemann Mapping theorem** in Page 7 is to input

```
\newtheoremstyle{newdiffnotenonum}{}{}%
  {\itshape}{}{\bfseries}{.}{ }%
    {\thmname{#1}\thmnote{ (\mdseries #3)}}

\theoremstyle{newdiffnotenonum}
\newtheorem{newRiemann}{Riemann Mapping Theorem}

\begin{newRiemann}
Every open simply connected proper subset of
$\mathbb{C}$ is analytically homeomorphic to the
open unit disk in $\mathbb{C}$.
\end{newRiemann}
```

Then you can also produce **Cauchy’s Theorem** in Page 6 by typing

```
\theoremstyle{newdiffnotenonum}
\newtheorem{newCuchy}{Cauchy’s Theorem}

\begin{newCauchy}[Third Version]
If $G$ is a simply connected open subset of $\mathbb{C}$, then for
every closed rectifiable curve $\gamma$ in $G$, we have
\begin{equation*}
 \int_\gamma f=0
\end{equation*}
\end{newCauchy}
```

The output will be exactly the same as that seen in Page 6. Now suppose you want to highlight certain theorems from other sources in your document, such as

Axiom 1 in (1). *Things that are equal to the same thing are equal to one another.*

This can be done as follows

```
\newtheoremstyle{citing}{}{}{\itshape}{}%
  {\bfseries}{.}{ }{\thmnote{#3}}

\theoremstyle{citing}
\newtheorem{cit}{}

\begin{cit}[Axiom 1 in \cite{eu}]
Things that are equal to the same thing are equal to one another.
\end{cit}
```

Of course, your *bibliography* should include the citation with *label* `eu`.

17.2.3 There’s more!

There are some more predefined features in amsthm package. In all the different examples we’ve seen so far, the *theorem number* comes after the *theorem name*. Some prefer to have it the other way round as in

17.2.1. Theorem (Euclid). *The sum of the angles in a triangle is* 180°

This effect is produced by the command `\swapnumbers` as shown below

```
\swapnumbers
\theoremstyle{plain}
\newtheorem{numfirstthm}{Theorem}[section]

\begin{numfirstthm}[Euclid]
The sum of the angles in a triangle is $180^\circ$
\end{numfirstthm}
```

Note that the `\swapnumbers` command is sort of toggle-switch, so that once it is given, *all subsequent theorem-like statements* will have their numbers first. If you want it the other way for some other theorem, then give `\swapnumbers` again before its definition.

A quick way to suppress *theoremnumbers* is to use the `\newtheorem*` command as in

```
\newtheorem*{numlessthm}{Theorem}[section]

\begin{numlessthm}[Euclid]
The sum of the angles in a triangle is $180^$.
\end{numlessthm}
```

to produce

Euclid. *The sum of the angles in a triangle is* 180°.

Note that this could also be done by leaving out `#2` in the *custom-head-spec* parameter of `\newtheoremstyle`, as seen earlier.

We've been talking only about *theorems* so far, but Mathematicians do not live by theorems alone; they need *proofs*. The amsthm package contains a predefined `proof` environment so that the proof of a theorem-like statement can be enclosed within `\begin{proof}...\end{proof}` commands as shown below

```
\begin{thmsec}
The number of primes is infinite
\end{thmsec}

\begin{proof}
Let $\{p_1,p_2,\dotsc p_k\}$ be a finite set of primes. Define
$n=p_1p_2\dotsm p_k+1$. Then either $n$ itself is a prime or has
a prime factor. Now  $n$ is neither equal to nor is divisible by
any of the primes $p_1,p_2,\dotsc p_k$ so that in either case,
we get a prime different from $p_1,p_2\dotsc p_k$. Thus no finite
set of primes includes all the primes.
\end{proof}
```

to produce the following output

Theorem 17.2.3. *The number of primes is infinite*

Proof. Let $\{p_1, p_2, \dotsc p_k\}$ be a finite set of primes. Define $n = p_1p_2 \cdots p_k + 1$. Then either n itself is a prime or has a prime factor. Now n is neither equal to nor is divisible by any of the primes $p_1, p_2, \dotsc p_k$ so that in either case, we get a prime different from $p_1, p_2 \dots p_k$. Thus no finite set of primes can include all the primes. □

There is an optional argument to the `proof` environment which can be used to change the *proofhead*. For example,

```
\begin{proof}[\textsc{Proof\,(Euclid)}:]
Let $\{p_1,p_2,\dotsc p_k\}$ be a finite set of primes. Define
$n=p_1p_2\dotsm p_k+1$. Then either $n$ itself is a prime or has
a prime factor. Now  $n$ is neither equal to nor is divisible by
any of the primes $p_1,p_2,\dotsc p_k$ so that in either case, we
get a prime different from $p_1,p_2\dotsc p_k$. Thus no finite
set of primes includes all the primes.
\end{proof}
```

produces the following

Proof (Euclid): Let $\{p_1, p_2, \dotsc p_k\}$ be a finite set of primes. Define $n = p_1 p_2 \dotsm p_k + 1$. Then either n itself is a prime or has a prime factor. Now n is neither equal to nor is divisible by any of the primes $p_1, p_2, \dotsc p_k$ so that in either case, we get a prime different from $p_1, p_2 \dotsc p_k$. Thus no finite set of primes can include all the primes. □

Note that the end of a proof is *automatically* marked with a □ which is defined in the package by the command `\qedsymbol`. If you wish to change it, use `\renewcommand` to redefine the `\qedsymbol`. Thus if you like the original "Halmos symbol" ▮ to mark the ends of your proofs, include

```
\newcommand{\halmos}{\rule{1mm}{2.5mm}}
\renewcommand{\qedsymbol}{\halmos}
```

in the preamble to your document.

Again, the placement of the `\qedsymbol` at the *end* of the last line of the proof is done via the command `\qed`. The default placement may not be very pleasing in some cases as in

Theorem 17.2.4. *The square of the sum of two numbers is equal to the sum of their squares and twice their product.*

Proof. This follows easily from the equation

$$(x + y)^2 = x^2 + y^2 + 2xy$$

□

It'd be better if this is typeset as

Theorem 17.2.5. *The square of the sum of two numbers is equal to the sum of their squares and twice their product.*

Proof. This follows easily from the equation

$$(x + y)^2 = x^2 + y^2 + 2xy \tag*{□}$$

which is achieved by the input shown below:

```
\begin{proof}
This follows easily from the equation
\begin{equation}
  (x+y)^2=x^2+y^2+2xy\tag*{\qed}
\end{equation}
\renewcommand{\qed}{} \end{proof}
```

For this trick to work, you must have loaded the package `amsmath` *without* the `leqno` option. Or, if you prefer

Proof. This follows easily from the equation

$$(x + y)^2 = x^2 + y^2 + 2xy \quad \square$$

Then you can use

```
\begin{proof}
This follows easily from the equation
\begin{equation*}
  (x+y)^2=x^2+y^2+2xy\qed
\end{equation*}
\renewcommand{\qed}{}
\end{proof}
```

17.3 Housekeeping

It's better to keep all `\newtheoremstyle` commands in the preamble than scattering them all over the document. Then you can divide your `\newtheorem` commands into groups and preface each group with the appropriate `\theoremstyle`.

Moreover, you can keep all your `\newtheoremstyle` in a `.thm` file, for instance `mystyles.thm`, and load it on demand in various documents using

```
\usepackage[mystyles]{amsthm}
```

This method fails if the `amsthm` has already been loaded by the documentclass such as `amsart`. In that case, you will have to use

```
\PassOptionsToPackage{mystyles}{amsthm}
```

Have fun!

References

[1] Euclid, *The Elements*, Greece 300 BC