Typesetting actuarial symbols easily and consistently with `actuarialsymbol` and `actuarialangle`

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Abstract

Actuarial notation is characterized by subscripts and superscripts on both sides of a principal symbol, numbers positioned above or below subscripts, and some otherwise unusual symbols. The pair of packages `actuarialsymbol` and `actuarialangle` provides all the facilities to compose actuarial symbols of life contingencies and financial mathematics, easily and consistently.

1 Introduction

Actuaries, the "engineers of insurance", denote various quantities of life contingencies using a whole array of symbols. The highly descriptive, yet compact, notation was standardized as far back as in 1898 [10]. Figure 1 shows a creative use of the notation by the graduating class of 1972 in Actuarial Science at Université Laval.

As most readers of *TUGboat* are probably unfamiliar with actuarial notation, let us start with the following examples:

1. the net single premium for an n-year term insurance payable at the end of year of death issued to a person aged x is $A^1_{x:\overline{n}|}$;
2. the monthly premium for an annual life annuity payable at the beginning of the year, starting n years from now is $P^{(12)}({}_{n|}\ddot{a}_x)$;
3. the net reserve at time t for a whole life insurance payable at death is ${}_t\bar{V}(\bar{A}_x)$.

All symbols are for nominal benefits of 1.

Actuarial notation is characterized by auxiliary symbols positioned in subscript and superscript on both sides of a principal symbol, something notoriously difficult to achieve consistently in LaTeX. It also requires some unusual symbols not found in standard mathematics packages, like the "angle" denoting a duration n, as in $\overline{n}|$, or the overhead angle bracket $\overline{xy}$ used to emphasize the joint status of lives x and y when ambiguity is possible.

The package `actuarialsymbol` [1] provides a generic command to position all subscripts and superscripts easily and consistently around a principal symbol, four commands to position precedence numbers above and below statuses, and a number of shortcuts to ease entry of the most common actuarial functions of financial mathematics and life contingencies. The companion package `actuarialangle` [3], separate from `actuarialsymbol` for historical rea-
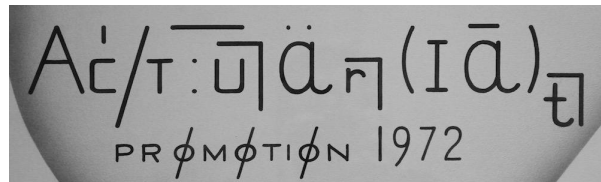
Figure 1: "Actuariat" (French for Actuarial Science) written using actuarial symbols on the 1972 graduating class mosaic at Université Laval

sons but imported by the latter, provides the angle and overhead angle bracket symbols.

2 Existing alternatives

Authors often use ad hoc constructions like `{}_tA_x` to put subscripts and superscripts in front of a symbol. This notation quickly becomes a nightmare to parse mentally and the source code has little relationship to the actual significance of the symbol. That said, the worst practical drawback to this approach is probably that there is no way to ensure that subscripts and superscripts on either side of the principal symbol are aligned vertically.

The package `mathtools` [5] provides a command `\prescript` to put a subscript or superscript to the left of an argument. This works well when the argument (or principal symbol) has sub- and superscripts on all four corners, but otherwise the auxiliary symbols may end up at different heights.

Finally, various packages tailored for specific disciplines offer the possibility to position sub- and superscripts on the left, for example `tensor` [7] for tensors or `mhchem` [4] for isotopes. There was a previous attempt at a LaTeX package for actuarial notation [9], but `lifecon` does not seem to be officially distributed, either from CTAN or from anywhere else.

3 Actuarial notation

Appendix 4 of [2] offers an excellent overview of the composition rules for symbols of actuarial functions. In a nutshell, a principal symbol, say S, is combined with auxiliary symbols positioned in subscript or in superscript, to the left or to the right. Schematically, we thus have:

$$\boxed{\frac{\boxed{\text{II}}}{\boxed{\text{I}}}\ S\ \frac{\boxed{\text{IV}}}{\boxed{\text{III}}}} \tag{1}$$

The principal symbol is in general a single letter. The letter may be "accented" with a bar ($\bar{A}$), double dots ($\ddot{a}$) or a circle ($\mathring{e}$). Most commonly, there are alphanumeric statuses in the lower-right position $\boxed{\text{III}}$. Numerals can be placed above or below the individual statuses to show the order of failure; we will refer to these numerals as *precedence numbers*.

Otherwise, auxiliary symbols appear lower-left $\boxed{\text{I}}$, upper-left $\boxed{\text{II}}$ and upper-right $\boxed{\text{IV}}$, in that order of frequency.

Symbols for benefit premiums (P), reserves (V) and amount of reduced paid-up insurance (W), are combined with benefit symbols unless the benefit is a level unit insurance payable at the end of the year of death. In such cases, we have the following symbol structure (replace P by V or W as needed):

$$\boxed{\substack{\boxed{\text{II}} \\ \boxed{\text{I}}}}\, P\, \boxed{^{\text{IV}}}(\boxed{S}\,\boxed{\text{III}}) \qquad (2)$$

4 Additional special symbols

The package `actuarialangle` defines commands to draw two special symbols used in actuarial and financial notation. In math mode, the command

`\angl{⟨duration⟩}`

composes an angle symbol around ⟨*duration*⟩ with some space (thin by default) between ⟨*duration*⟩ and the right descender. The symbol scales gracefully if the command is used outside of a first-level subscript.

`\angl{n} \quad a_{\angl{n}}`	$\overline{n}|$ $\quad$ $a_{\overline{n}|}$

Commands `\angln`, `\anglr` and `\anglk` are shortcuts for the common cases `\angl{n}`, `\angl{r}` and `\angl{k}`, respectively.

The code for `\angl` and the underlying macro were given to the second author by a colleague many years ago. The original author is unknown.

The command

`\overanglebracket{⟨statuses⟩}`

composes an angle bracket ("roof") above ⟨*statuses*⟩. The rule thickness and spacing relative to the statuses match those of the angle symbol. The command `\group` is a convenient alias for `\overanglebracket`.

`\group{xy} \quad` `    A_{\group{xy}:\angln}`	$\overline{xy}$ $\quad$ $A_{\overline{xy}:\overline{n}|}$

5 Construction of actuarial symbols

The package `actuarialsymbol` provides the generic command `\actsymb` to typeset a principal symbol with surrounding subscripts and superscripts. Its syntax is somewhat unusual for LaTeX, but it serves well the natural order of the building blocks of a symbol and their relative prevalence:

`\actsymb[⟨ll⟩][⟨ul⟩]{⟨symbol⟩}{⟨lr⟩}[⟨ur⟩]`

Above, ⟨*ll*⟩ identifies the auxiliary symbol in the lower left subscript position $\boxed{\text{I}}$ (following the notation in the schematic representation (1)); ⟨*ul*⟩ is the upper left superscript $\boxed{\text{II}}$; ⟨*symbol*⟩ is the principal symbol S; ⟨*lr*⟩ is the lower right subscript $\boxed{\text{III}}$; and

⟨*ur*⟩ is the upper right superscript $\boxed{\text{IV}}$. The principal symbol and the right subscript are required, the other arguments are optional.

`\actsymb{A}{x}`	A_x
`\actsymb[n]{A}{x}`	$_nA_x$
`\actsymb[n][2]{A}{x}`	$_n^2A_x$
`\actsymb[n][2]{A}{x}[(m)]`	$_n^2A_x^{(m)}$

The command `\actsymb` supports one more optional argument, for composing symbols for premiums, reserves and paid-up insurance. The extended command

`\actsymb[⟨ll⟩][⟨ul⟩][⟨P⟩]{⟨symbol⟩}{⟨lr⟩}[⟨ur⟩]`

puts the symbol ⟨*P*⟩ outside the parentheses in the schematic representation (2).

`\actsymb[][][P]{\bar{A}}{x:\angln}`	$P(\bar{A}_{x:\overline{n}|})$
`\actsymb[k][][V]{\bar{A}}{x}[\{1\}]`	$_kV^{\{1\}}(\bar{A}_x)$
`\actsymb[k][][\bar{W}]{\bar{A}}{x}`	$_k\bar{W}(\bar{A}_x)$

Composing actuarial symbols from scratch using `\actsymb` can easily get quite involved. For this reason, the package defines a large number of shortcut macros to ease entry of the most common symbols. Table 1 offers a glimpse of the available shortcuts; the package documentation has the complete list.

The definition of `\actsymb` is heavily inspired by the code of `\prescript` from package `mathtools` which, as reported by the author, is itself based on a posting to `comp.text.tex` by Michael J. Downes.

6 Positioning of subscripts

TeX adjusts the position of a subscript downward when a superscript is present:

$$A_x \qquad A_x^2.$$

Command `\actsymb` maintains this behavior, something we believe to be a desirable feature. Therefore, entering the symbols above using the standard operators `^` and `_` or with `\actsymb` yields the same result.

`A_x \quad A_x^2`	A_x	A_x^2
`\actsymb{A}{x} \quad` `    \actsymb{A}{x}[2]`	A_x	A_x^2

Furthermore, the command ensures that the left and right subscripts, when both present, are at the same level, something common ad hoc constructions do not provide.

`{}_tA_x \quad {}_tA_x^2`	$_tA_x$	$_tA_x^2$
`\actsymb[t]{A}{x} \quad` `    \actsymb[t]{A}{x}[2]`	$_tA_x$	$_tA_x^2$

Authors who would prefer a uniform subscript position *throughout their document* can load the package `subdepth` [8].

Table 1: Sample of shortcuts for life table, insurance and annuity symbols.
All commands accept the optional arguments $\langle ll \rangle$, $\langle ul \rangle$ and $\langle ur \rangle$ of \actsymb.

Definition	Example	Output
\lx{⟨*age*⟩}	\lx{x}	ℓ_x
\dx{⟨*age*⟩}	\dx[n]{x}	$_n d_x$
\px{⟨*age*⟩}	\px[t]{x}	$_t p_x$
\qx{⟨*age*⟩}	\qx[t]{x}	$_t q_x$
\eringx{⟨*lr*⟩}	\eringx{x:\angln}	$\mathring{e}_{x:\overline{n}|}$
\Ax{⟨*lr*⟩}	\Ax{x:\angln}	$A_{x:\overline{n}|}$
\Ax*{⟨*lr*⟩}	\Ax*{x:\angln}	$\bar{A}_{x:\overline{n}|}$
\Ex{⟨*lr*⟩}	\Ex[n]{x}	$_n E_x$
\ax{⟨*lr*⟩}	\ax{x:\angln}	$a_{x:\overline{n}|}$
\ax*{⟨*lr*⟩}	\ax*{x:\angln}	$\bar{a}_{x:\overline{n}|}$
\ax**{⟨*lr*⟩}	\ax**{x:\angln}	$\ddot{a}_{x:\overline{n}|}$
\aringx{⟨*lr*⟩}	\aringx{x:\angln}	$\mathring{a}_{x:\overline{n}|}$

7 Precedence numbers

Precedence numbers appear above or below individual statuses in the right subscript ⌐III¬ of a symbol. The commands

\nthtop[⟨*length*⟩]{⟨*number*⟩}{⟨*status*⟩}
\nthbottom[⟨*length*⟩]{⟨*number*⟩}{⟨*status*⟩}

put a precedence ⟨*number*⟩ above (resp. below) a ⟨*status*⟩, smashed so that the apparent height of the status is its normal height.

\actsymb{A}{\nthtop{1}{x}:\angln}	$A_{x:\overline{n}|}^1$
\actsymb{A}{x:\nthtop{1}{\angln}}	$A_{x:\frac{1}{n}|}$
\actsymb{A}{\nthtop{1}{x}y:% \nthtop{2}{\angln}}	$A_{xy:\overline{n}|}^{1\ 2}$
\actsymb{A}{\nthtop{3}{x}% \nthbottom{1}{y}\nthbottom{2}{z}}	$A_{xyz\atop 12}^3$

As can be seen in the third and fourth examples above, the constant spacing between the precedence number and the status can result in numbers placed at different heights if one status contains a horizontal rule or a descender. To cope with this situation, we provide * variants of the commands that always align precedence numbers vertically.

\actsymb{A}{\nthtop*{1}{x}y:% \nthtop*{2}{\angln}}	$A_{xy:\overline{n}|}^{1\ 2}$
\actsymb{A}{\nthtop{3}{x}% \nthbottom*{1}{y}\nthbottom*{2}{z}}	$A_{xyz\atop 12}^3$

The fact that top precedence numbers have zero height means they will clash with a right superscript ⌐IV¬ .

\actsymb{A}{\nthtop{1}{x}:\angln}[(m)]	$A_{x:\overline{n}|}^{(m)}$

For such rare circumstances, we left to the user to insert a strut in the subscript to push it downward as needed.

\actsymb{A}{\rule{0pt}{2.3ex}% \nthtop{1}{x}:\angln}[(m)]	$A_{x:\overline{n}|}^{(m)}$

This remark also applies to bottom precedence numbers in inline formulas or multiline equations.

The optional argument ⟨*length*⟩ of \nthtop and \nthbottom changes the default spacing between the number and the status for one symbol. This can also be changed globally by redefining lengths mentioned in the documentation of actuarialsymbol.

The package defines shortcuts \itop, \iitop and \iiitop for first, second and third top precedence (and their analogues for bottom precedence).

The system of precedence numbers builds on a macro that used to be part of actuarialangle. As with the code for \angl, the original author is unknown.

8 Other functionalities

For brevity, we have omitted some additional features of actuarialsymbol, including macros to typeset two-letter symbols such as (IA), numerous shortcut macros and quite fancy utilities to define new ones. The package documentation provides all the details.

Following [9], the package documentation also contains a *Comprehensive list of life contingencies symbols*. The wording used here should be taken for its intended purpose, namely to acknowledge Scott Pakin's immensely useful *Comprehensive LaTeX Symbol List* [6].

References

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